

Gravity-induced entropy in the quantum motion of a macroscopic

body.

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It is shown that a recently proposed model for the gravitational interaction in non relativistic quantum mechanics may turn to be relevant to the derivation of the second law of thermodynamics. In particular, the spreading of the probability density of the center of mass of an isolated macroscopic body does not imply delocalization of the wave function, but on the contrary it corresponds to an entropy growth.

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The monotone time dependence of the entropy of an isolated system and the emergence of the arrow of time have long been staying as a debated issue, since the birth of classical statistical mechanics. A brief sketch of some contributions to that long-standing debate within quantum physics [1–6] can be found in Ref. [7], where a quantum approach to the derivation of the second law of thermodynamics was proposed. In it a non unitary dynamics of a gas-system ensues from the peculiarity of the model, where the interaction with the container obeys a suitable form of generalized microcanonicity, namely its only effect is to produce entanglement, without affecting energy conservation.

In this letter we want to explore the possibility that even for a genuinely isolated system an entropy growth takes place, within a non relativistic quantum description, owing to the gravitational interaction. In order to do that we use a model for the gravitational interaction

in non relativistic quantum mechanics [8], where self-interactions are treated on the same footing as mutual interactions between different bodies. This is achieved by duplicating the ordinary degrees of freedom, with the introduction of a (red) partner for every ordinary (green) particle and of a Newton gravitational interaction between (meta)particles of different color only, and restricting the (meta)state space by a suitable constraint. Once red metaparticles are traced out, the resulting non unitary dynamics reproduces both the classical aspects of the gravitational interaction and wave function localization of macroscopic bodies [8].

Even in a one particle model like the Schroedinger-Newton theory, without added unobservable degrees of freedom, one can exhibit stationary localized states [9]. However, apart from the price paid in abandoning the traditional linear setting of QM, they sound quite unrealistic, as the initial linear momentum uncertainty is expected to lead to a spreading of the probability density in space. On the other hand a theory of wave function localization has to keep localization during time evolution. Our model offers a way out of this apparent paradox, leading to a spreading that consists in the emergence of delocalized ensembles of localized pure states.

To be specific, let $H[\psi^\dagger, \psi]$ denote, following Ref. [8], the second quantized non-relativistic Hamiltonian of a finite number of particle species, like electrons, nuclei, ions, atoms and/or molecules, according to the energy scale. For notational simplicity ψ^\dagger, ψ denote the whole set $\psi_j^\dagger(x), \psi_j(x)$ of creation-annihilation operators, i.e. one couple per particle species and spin component. This Hamiltonian includes the usual electromagnetic interactions accounted for in atomic and molecular physics. To incorporate gravitational interactions including self-interactions, we introduce complementary creation-annihilation operators $\chi_j^\dagger(x), \chi_j(x)$ and the overall Hamiltonian

$$H_G = H[\psi^\dagger, \psi] + H[\chi^\dagger, \chi] - G \sum_{j,k} m_j m_k \int dx dy \frac{\psi_j^\dagger(x) \psi_j(x) \chi_k^\dagger(y) \chi_k(y)}{|x - y|}, \quad (1)$$

acting on the tensor product $F_\psi \otimes F_\chi$ of the Fock spaces of the ψ and χ operators, where m_i denotes the mass of the i -th particle species and G is the gravitational constant. While

the χ operators are taken to obey the same statistics as the original operators ψ , we take advantage of the arbitrariness pertaining to distinct operators and, for simplicity, we choose them commuting with one another: $[\psi, \chi]_- = [\psi, \chi^\dagger]_- = 0$.

The metaparticle state space S is identified with the subspace of $F_\psi \otimes F_\chi$ including the metastates obtained from the vacuum $|0\rangle = |0\rangle_\psi \otimes |0\rangle_\chi$ by applying operators built in terms of the products $\psi_j^\dagger(x)\chi_j^\dagger(y)$ and symmetrical with respect to the interchange $\psi^\dagger \leftrightarrow \chi^\dagger$, which, as a consequence, have the same number of ψ (green) and χ (red) metaparticles of each species. In particular the most general metastate corresponding to one j -particle states is represented by

$$||f\rangle\rangle = \int dx \int dy f(x, y) \psi_j^\dagger(x) \chi_j^\dagger(y) |0\rangle, \quad f(x, y) = f(y, x), \quad (2)$$

with one green and one red j -metaparticle. This is a consistent definition since the overall Hamiltonian is such that the corresponding time evolution is a group of (unitary) endomorphisms of S . If we prepare a pure n -particle state, represented in the original setting - excluding gravitational interactions - by

$$|g\rangle \doteq \int d^n x g(x_1, x_2, \dots, x_n) \psi_{j_1}^\dagger(x_1) \psi_{j_2}^\dagger(x_2) \dots \psi_{j_n}^\dagger(x_n) |0\rangle, \quad (3)$$

its representation in S is given by the metastate

$$||g \otimes g\rangle\rangle = \int d^n x d^n y g(x_1, \dots, x_n) g(y_1, \dots, y_n) \psi_{j_1}^\dagger(x_1) \dots \psi_{j_n}^\dagger(x_n) \chi_{j_1}^\dagger(y_1) \dots \chi_{j_n}^\dagger(y_n) |0\rangle. \quad (4)$$

As for the physical algebra, it is identified with the operator algebra of say the green metaworld. In view of this, expectation values can be evaluated by preliminarily tracing out the χ operators and then taking the average in accordance with the traditional setting.

While we are talking trivialities as to an initial metastate like in Eq. (4), that is not the case in the course of time, since the gravitational interaction in the overall Hamiltonian produces entanglement between the two metaworlds, leading, once χ operators are traced out, to mixed states of the physical algebra. It was shown in Ref. [8] that the ensuing non-unitary evolution induces both an effective interaction mimicking gravitation, and wave function localization.

It was shown also that, omitting the internal wave function, a localized metastate of an isolated homogeneous spherical macroscopic body of radius R and mass M can be represented by

$$\tilde{\Psi}_0(X, Y) \propto \exp \frac{-|X - Y|^2}{2\Lambda^2} \exp \frac{-|X + Y|^2}{2\Lambda^2}, \quad \Lambda^2 = \frac{\hbar}{\sqrt{\alpha G M^3 / R^3}}, \quad (5)$$

where $\alpha \sim 10^0$ and G is the gravitational constant. In Eq. (5) X and Y are respectively the position of the center of mass of the green and the red metabody. The first factor is proportional to the wave function of the relative motion and, for bodies of ordinary density $\sim 1 \text{ gm/cm}^3$ and whose mass exceeds $\sim 10^{11}$ proton masses, it represents its ground state [8]. The second factor is proportional to the wave function of the center of metamass $(X + Y)/2$ and spreads in time as usual for the free motion of the center of mass of a body of mass $2M$, so that after a time t , in the absence of external forces, the metawave function becomes

$$\tilde{\Psi}_t(X, Y) \propto \exp \frac{-|X - Y|^2}{2\Lambda^2} \exp \frac{-|X + Y|^2/4}{\Lambda^2/2 + i\hbar t/M} \equiv \exp [-\alpha_0 |X - Y|^2] \exp [-\alpha_t |X + Y|^2]. \quad (6)$$

In order that this may be compatible with the assumption that gravity continuously forces localization, the spreading of the physical state must be the outcome of a growth of the corresponding entropy. This initially vanishes, since the initial metawave function is unentangled:

$$\tilde{\Psi}_0(X, Y) \propto \exp \frac{-X^2}{\Lambda^2} \exp \frac{-Y^2}{\Lambda^2}, \quad (7)$$

and then the corresponding physical state obtained by tracing out Y is pure. If one evaluates the physical state according to

$$\rho_t(X, X') = \int dY \tilde{\Psi}_t(X, Y) \tilde{\Psi}_t^*(X', Y), \quad (8)$$

one finds that the space probability density is given by

$$\rho_t(X, X) = \left[\frac{8\alpha_0(\alpha_t + \bar{\alpha}_t)}{\pi(\alpha_t + \bar{\alpha}_t + 2\alpha_0)} \right]^{3/2} \exp \left[-\frac{8\alpha_0(\alpha_t + \bar{\alpha}_t)}{(\alpha_t + \bar{\alpha}_t + 2\alpha_0)} X^2 \right] \propto \exp \frac{-2\Lambda^2 X^2}{\Lambda^4 + 2\hbar^2 t^2 / M^2} \quad (9)$$

Parenthetically it is worth while to remark that this spreading of the probability density is slower than the one ensuing from the spreading of the wave function in the absence of the gravitational self-interaction, which leads to

$$\rho_t(X, X) \propto \exp \frac{-2\Lambda^2 X^2}{\Lambda^4 + 4\hbar^2 t^2 / M^2}, \quad (10)$$

and that both are extremely slow, as their typical time, for macroscopic bodies of ordinary density, is $\sim 10^3$ sec independently from the mass, as can be checked by means of Eqs. (9) and (10).

If this spreading is due to entropy growth only, rather than to the usual spreading of the wave function, the corresponding entropy S_t is expected to depend approximately on the ratio between the final and the initial space volumes roughly occupied by the two gaussian densities, according to

$$S_t \sim K_B \frac{3}{2} \ln \left[\frac{\alpha_t + \bar{\alpha}_t + 2\alpha_0}{2(\alpha_t + \bar{\alpha}_t)} \right], \quad (11)$$

at least for large enough times. (Linear momentum probability density does not depend on time.) Of course this corresponds to approximating the mixed state by means of an ensemble of N equiprobable localized states, which is legitimate if N is large enough. In order to evaluate the entropy of the state represented by $\rho_t(X, X')$ and to check Eq. (11), we use the possibility, in this approximation, of linking the entropy

$$S_t = -K_B \text{Tr} [\rho_t \ln \rho_t] = K_B \ln N \quad (12)$$

with the purity

$$\text{Tr} [\rho_t^2] = \frac{1}{N}, \quad (13)$$

where of course

$$\rho_t^2(X, X') = \int dX'' \rho_t(X, X'') \rho_t(X'', X'). \quad (14)$$

By an explicit computation we get

$$Tr [\rho_t^2] = \int dX \rho_t^2(X, X) = \frac{[4\alpha_0(\alpha_t + \bar{\alpha}_t)]^3}{\left[(2\alpha_t\bar{\alpha}_t + 6\alpha_t\alpha_0 + 6\bar{\alpha}_t\alpha_0 + 2\alpha_0^2)^2 - 4(\bar{\alpha}_t - \alpha_0)^2(\alpha_t - \alpha_0)^2\right]^{3/2}}, \quad (15)$$

and, for large times, namely small α_t , one can keep in this result just the leading term in α_t , that is

$$Tr [\rho_t^2] \sim \left(\frac{\alpha_t + \bar{\alpha}_t}{2\alpha_0}\right)^{3/2}, \quad (16)$$

which, by using Eqs. (12,13), gives

$$S_t \sim -K_B \frac{3}{2} \ln \left(\frac{\alpha_t + \bar{\alpha}_t}{2\alpha_0}\right) = K_B \frac{3}{2} \ln \left(\frac{\Lambda^4 + 4\hbar^2 t^2 / M^2}{\Lambda^4}\right), \quad (17)$$

which differs from the leading term in Eq.(11) by an irrelevant quantity $(3/2)K_B \ln 2$.

It is worth while to remark that, while the present model is expected to be just a low energy approximation to a conceivable more general theory, its present application to a free motion is expected basically to reproduce the possible exact outcome of the latter. In fact the analysis refers to the rest frame of the probability density and implies exceedingly small velocities, as can be checked by taking the Fourier transform of the wave function in Eq. (6).

This result is an encouraging hint towards the possibility of deriving the second law of thermodynamics for genuinely isolated systems from what most people call quantum gravity, or say from quantum mechanics with a proper inclusion of gravity. Of course much remains to be done both with reference to the consideration of further instances, physically more comprehensive than just free motions of macroscopic bodies, and to the extension of the present approach, to cope with energies higher than the ones where the non relativistic model is appropriate.

Finally it should be made clear that the present letter addresses what can be called fundamental entropy growth, which in principle could be defined even for the universe as a whole. For real, only approximately isolated, systems one would expect that usually, however small the coupling with the environment may be, the corresponding entanglement entropy [7] is

easily large enough to overshadow the fundamental one. This is the thermodynamical counterpart of the overshadowing of fundamental decoherence [8] by the environment induced one [10]. Just as for this latter issue one may expect that, while waiting for a possible future experimental detection, fundamental decoherence may play a role with reference to the measurement problem in QM, likewise fundamental entropy growth is a natural candidate to address the issue of the arrow of time. While the entropy growth we are talking about is extremely small, that is only natural for a possible breaking of time reversal symmetry responsible for the time arrow. This smallness, on the other hand, is connected with the high localization threshold at $\sim 10^{11}$ proton masses, amply compatible with the present lower bounds at $\sim 10^3$ proton masses [11,12].

To sum up, for a generic isolated system the present picture leads to surmise that time flow is characterized by the monotone increase of the entanglement between observable and unobservable degrees of freedom. While this view can be taken in principle within more conventional attempts to build a quantum theory of gravity as well [13], a peculiar feature of the present approach is the simple and unambiguous definition of the unobservable degrees of freedom and of the overall dynamics, which makes the model a viable computational tool.

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